

# Dark Energy and Dark Matter in Galaxy Halos

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## Abstract

We consider the possibility that the dark matter is coupled through its mass to a scalar field associated with the dark energy of the Universe. In order for such a field to play a role at the present cosmological distances, it must be effectively massless at galactic length scales. We discuss the effect of the field on the distribution of dark matter in galaxy halos. We show that the profile of the distribution outside the galaxy core remains largely unaffected and the approximately flat rotation curves persist. The dispersion of the dark matter velocity is enhanced by a potentially large factor relative to the case of zero coupling between dark energy and dark matter. The counting rates in terrestrial dark matter detectors are similarly enhanced. Existing bounds on the properties of dark matter candidates can be extended to the coupled case, by taking into account the enhancement factor.

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According to our present understanding of the matter content of the Universe, its energy density is dominated by dark energy and dark matter. The usual assumption is that the dark energy is associated with the potential of an evolving scalar field [1, 2], while the dark matter is composed of weakly interacting massive particles. Usually these two sectors are assumed to interact only gravitationally. Attempts to resolve the coincidence problem (the comparable present contributions to the total energy density from the two components) are based on the presence of an additional interaction between the dark matter and the scalar field [3]. The cosmological evolution now depends on the potential of the field, as well as on the type and strength of the interaction [4]. For a scalar field to be relevant for the cosmological evolution today, so as to resolve the coincidence problem, the scale for its effective mass must be set by the present value of the Hubble parameter [3]. This means that the field is effectively massless at galactic length scales. Its coupling to the dark matter particles results in a long-range force that can affect the details of structure formation [5, 6].

The presence of a long-range scalar interaction in addition to gravity can modify the distribution of dark matter in galaxy halos. In particular, it is not certain that the dark matter distribution can maintain a form that results in approximately flat rotation curves for objects orbiting galaxies. Recent simulations of structure formation [6] are not detailed enough at the scales of interest in order to resolve this question.

We consider an interaction between the scalar field and the dark matter particles that can be modelled through a field-dependent particle mass. The action takes the form

$$\mathcal{S} = \int d^4x \sqrt{-g} \left( M^2 R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - U(\phi) \right) - \sum_i \int m(\phi(x_i)) d\tau_i + \mathcal{S}_b, \quad (1)$$

where  $d\tau_i = \sqrt{-g_{\mu\nu}(x_i) dx_i^\mu dx_i^\nu}$  and the second integral is taken over particle trajectories. The contribution  $\mathcal{S}_b$  describes the standard baryonic component. Variation of the action with respect to  $\phi$  results in the equation of motion

$$\frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu \phi) = \frac{dU}{d\phi} - \frac{d \ln m(\phi(x))}{d\phi} T^\mu_\mu, \quad (2)$$

where the energy-momentum tensor associated with the gas of particles is

$$T^{\mu\nu} = \frac{1}{\sqrt{-g}} \sum_i \int d\tau_i m(\phi(x_i)) \frac{dx_i^\mu}{d\tau_i} \frac{dx_i^\nu}{d\tau_i} \delta^{(4)}(x - x_i). \quad (3)$$

We are interested in static spherically symmetric configurations, with the scalar field varying slowly with the radial distance  $r$ . Our treatment is relevant up to a distance  $r_1 \sim 100$  kpc beyond which the dark matter becomes very dilute. For  $r \gtrsim r_1$  we expect that  $\phi$  quickly becomes constant with a value close to  $\phi(r_1) \equiv \phi_1$ . This is the value that drives the present cosmological expansion. Here we assume that the cosmological evolution of  $\phi_1$  is negligible for the time scales of interest, so that the asymptotic configuration is static to a good approximation.

We approximate:  $m(\phi) \simeq m(\phi_0) + [dm(\phi_0)/d\phi] \delta\phi \equiv m_0 + m'_0 \delta\phi$ , with  $\phi_0$  the value of the field at the center of the galaxy ( $r = 0$ ). We work within the leading order in  $\delta\phi$  and assume that  $m'/m \simeq m'_0/m_0$  for all  $r$ . Also  $dU/d\phi$  can be approximated by a constant between  $r = 0$  and  $r = \infty$ . For the scalar field to provide a resolution of the coincidence problem, the two terms in the r.h.s. of eq. (2) must be of similar magnitude in the cosmological solution. This

means that  $dU/d\phi$  must be comparable to  $(m'_0/m_0)\rho_\infty$ . We expect  $\rho_\infty$  to be a fraction of the critical density, i.e.  $\rho_\infty \sim 3 \text{ keV/cm}^3$ . On the other hand, the energy density in the central region of the static solution ( $r \lesssim 100 \text{ kpc}$ ) is that of the galaxy halo ( $\sim 0.4 \text{ GeV/cm}^3$  for our neighborhood of the Milky Way). This makes  $dU/d\phi$  negligible in the r.h.s. of eq. (2) for a static configuration. The potential is expected to become important only for  $r \rightarrow \infty$ , where the static solution must be replaced by the cosmological one. Similar arguments indicate that we can neglect  $U$  relative to  $\rho$ . Also the scalar field must be effectively massless at the galactic scale. For these reasons we expect that the form of the potential plays a negligible role at the galactic level. Our analysis can be carried out with  $U = 0$  and is model independent.

We treat the dark matter as a weakly interacting, dilute gas. We are motivated by the phenomenological success of the isothermal sphere [7] in describing the flat part of the rotation curves. We do not address the question of the density profile in the inner part of the galaxies ( $r \lesssim 5 \text{ kpc}$ ). We approximate the energy-momentum tensor of the dark matter as  $T^\mu_\nu = \text{diag}(-\rho, p, p, p)$  with  $p(r) = \rho(r) \langle v_d^2 \rangle = m(\phi(r)) n(r) \langle v_d^2 \rangle$ . The dispersion of the dark matter velocity is assumed to be constant and small:  $\langle v_d^2 \rangle \ll 1$ . The gravitational field is considered in the Newtonian approximation:  $g_{00} \simeq 1 + 2\Phi$ , with  $\Phi = \mathcal{O}(m'_0 \delta\phi/m_0)$ . In the weak field limit and for  $p \ll \rho$ , the conservation of the energy-momentum tensor gives

$$p' = -\rho \Phi' - \rho \frac{m'_0}{m_0} (\delta\phi)', \quad (4)$$

with the prime on  $p$ ,  $\Phi$ ,  $\delta\phi$  denoting a derivative with respect to  $r$ . Integration of this equation gives  $n \simeq n_0 \exp(-\Phi/\langle v_d^2 \rangle - (m'_0/m_0)\delta\phi/\langle v_d^2 \rangle)$ . In order to make the picture more complete we also allow for a pressureless baryonic component in the core. We assume that its energy density has the phenomenological profile  $\rho_b(r) = \rho_B f(r/r_c)$ , with  $f(x)$  a decreasing function of  $x$ . We do not discuss the physics that leads to such a profile. We include the baryonic contribution only in order to estimate its effect on the dark matter distribution in the outer regions of the galaxy.

With the above assumptions we obtain the equations of motion

$$\Phi'' + \frac{2}{r} \Phi' = \frac{1}{4M^2} \rho_0 \exp(-\alpha\Phi - \tilde{\alpha}\delta\phi) + \frac{1}{4M^2} \rho_B f\left(\frac{r}{r_c}\right), \quad (5)$$

and

$$(\delta\phi)'' + \frac{2}{r} (\delta\phi)' = \frac{m'_0}{m_0} \rho_0 \exp(-\alpha\Phi - \tilde{\alpha}\delta\phi), \quad (6)$$

where  $M = (16\pi G_N)^{-1/2}$  is the reduced Planck mass,  $\rho_0 = m_0 n_0$  the energy density of dark matter at  $r = 0$ ,  $\alpha = 1/\langle v_d^2 \rangle$ , and  $\tilde{\alpha} = m'_0/(m_0 \langle v_d^2 \rangle)$ . We emphasize that, even though  $|\Phi| \ll 1$ , the combination  $\Phi/\langle v_d^2 \rangle$ , that appears in the exponent in the expression for the number density  $n$ , can be large. Similarly, the expansion of the mass around the value  $m_0 = m(\phi_0)$  assumes the smallness of the dimensionless parameter  $|m'_0 \delta\phi/m_0|$ . However, the combination  $\tilde{\alpha} \delta\phi = (m'_0 \delta\phi/m_0)/\langle v_d^2 \rangle$ , that appears in the exponent, can be large.

A linear combination of eqs. (5), (6) gives

$$\frac{d^2 u}{dz^2} + \frac{2}{z} \frac{du}{dz} + \exp u + \frac{R}{1 + \kappa^2} f\left(\frac{z}{z_c}\right) = 0, \quad (7)$$

where  $u = -\alpha\Phi - \tilde{\alpha}\delta\phi$ ,  $z = \beta r$ ,  $z_c = \beta r_c$ ,  $R = \rho_B/\rho_0$ , and  $\beta^2 = (1 + \kappa^2)\alpha\rho_0/4M^2$ . The parameter  $\kappa^2 = 4M^2 (m'_0/m_0)^2$  determines the strength of the new interaction relative to gravity. For large

$z$  all regular solutions approach the form  $\exp u = 2/z^2$ . Another linear combination of eqs. (5), (6) gives

$$\frac{d^2 v}{dz^2} + \frac{2}{z} \frac{dv}{dz} + \frac{\kappa^2 R}{1 + \kappa^2} f\left(\frac{z}{z_c}\right) = 0, \quad (8)$$

with  $v = -\kappa^2 \alpha \Phi + \tilde{\alpha} \delta \phi$ . For large  $z$  the solution of this equation is  $v = c_0 + c_1/z$ . Combining the two solutions gives the leading result for large  $r$

$$\Phi' = \frac{2\langle v_d^2 \rangle}{1 + \kappa^2} \frac{1}{r}. \quad (9)$$

If the new interaction is universal for ordinary and dark matter, the experimental constraints impose  $\kappa^2 \ll 1$ . In this case, it is reasonable to expect a negligible effect in the distribution of matter in galaxy halos. However, if  $\phi$  interacts only with dark matter, as we assume here, this bound can be relaxed significantly.

According to eq. (9), a massive particle in orbit around the galaxy, at a large distance  $r$  from its center, has a velocity  $v_c^2 = 2\langle v_d^2 \rangle / (1 + \kappa^2)$ . We can use this expression in order to fix  $\langle v_d^2 \rangle$  for a given value of  $\kappa$ . The approximately flat rotation curves are a persistent feature even in the case that the dark matter is coupled to a scalar field through its mass. The effect of the new scalar interaction is encoded in the factor  $\kappa^2$ . When this is small, the velocity of an object orbiting the galaxy is of the order of the square root of the dispersion of the dark matter velocity. If  $\kappa^2$  is large, the rotation velocity can become much smaller than the typical dark matter velocity.

The Newtonian potential can be expressed as  $\Phi' = -2(u' + v')/v_c^2$ . This allows us to write the rotation velocity as

$$\left(\frac{v}{v_c}\right)^2 = \frac{r\Phi'}{v_c^2} = -\frac{z}{2} \left(\frac{du}{dz} + \frac{dv}{dz}\right). \quad (10)$$

We can also relate the parameters  $z$  and  $r$  through the expression

$$z = \beta r \simeq \left(\frac{\rho_0}{0.2 M_\odot/\text{pc}^3}\right)^{1/2} \frac{150 \text{ km/s}}{v_c} \frac{r}{\text{kpc}}. \quad (11)$$

In order to study the rotation velocity profile more quantitatively we integrate eqs. (7), (8) in the interval  $0 \leq z \leq 30$ . We normalize the Newtonian potential  $\Phi$  so that  $\Phi(0) = 0$ . Also, according to our definition,  $\delta\phi(r_0) = 0$ . These relations imply  $u(0) = v(0) = 0$ . The initial conditions  $du(0)/dz = dv(0)/dz = 0$  guarantee that the solution is regular at  $r = 0$ . In fig. 1 we present the solution of eqs. (7), (8) for various values of  $\kappa$  and  $R$ . In all cases we use  $f(x) = \exp(-x)$  and  $z_c = 0.5$ . We observe that the solutions do not differ substantially. Approximately flat rotation curves are expected for  $z \gtrsim 5$ , consistently with observations [7]. This behaviour persists for other forms of the profile of baryonic matter, as long as dark matter dominates over baryonic matter for  $z \gtrsim 3$ . The flat part of the rotation curves is largely insensitive to the structure of the core. The departure from spherical symmetry for the distribution of matter in the core of a spiral galaxy is also expected to be a minor effect.

Independently of the value of  $\kappa^2$ , the interaction of dark matter with the scalar field associated with dark energy does not destroy the approximately flat profile of the rotation curves. Other considerations, however, could constrain the coupling  $\kappa^2 = 4M^2(m'_0/m_0)^2$ . The dispersion of the dark matter velocity is  $\langle v_d^2 \rangle = (1 + \kappa^2)v_c^2/2$ . For a value of  $v_c$  deduced from observations,  $\langle v_d^2 \rangle$  increases with  $\kappa$ . For sufficiently large  $\kappa$ , it seems possible that  $v_c$  may exceed the escape velocity from the galaxy. It turns out, however, that this is not the case. Outside the

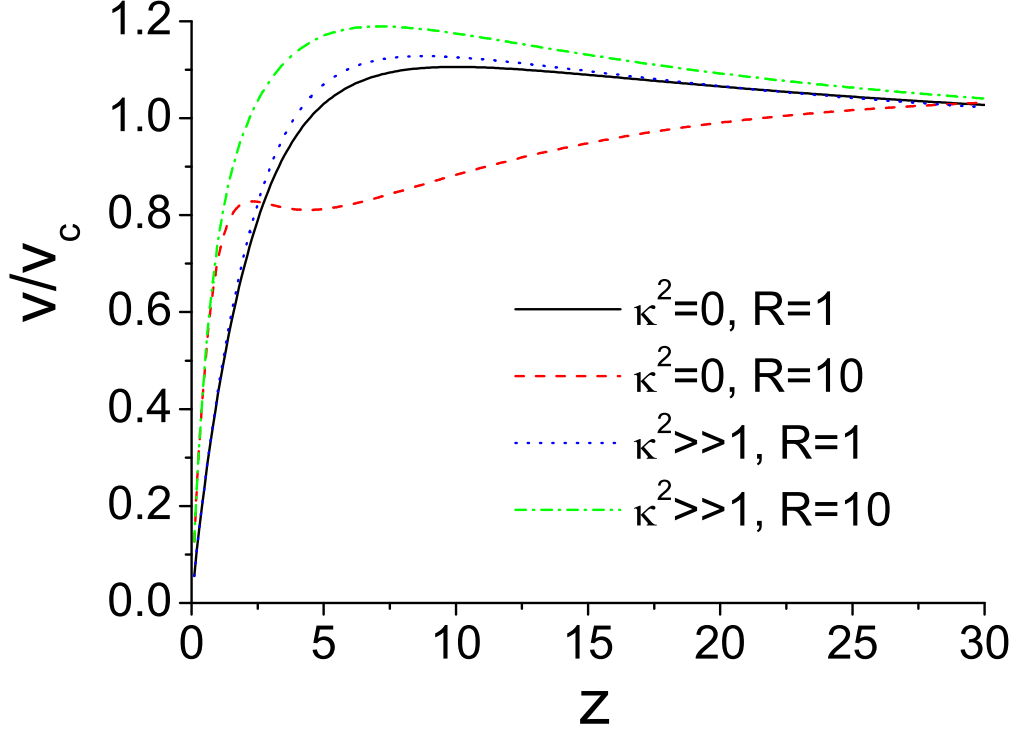


Figure 1: *The shape of the rotation curves for various values of  $\kappa$  and  $R$ .*

core of the galaxy and for  $r \lesssim r_1$ , the binding potential for a dark matter particle is  $\Phi + (\tilde{\alpha}/\alpha)\delta\phi$ . For large  $r$ , eq. (8) implies that  $v = -\kappa^2\alpha\Phi + \tilde{\alpha}\delta\phi = \text{constant}$ . The binding potential becomes  $(1+\kappa^2)\Phi = (1+\kappa^2)v_c^2 \ln(r/r_1)$ , where we have omitted an overall constant. For a particle at a distance  $r_*$  from the center of the galaxy, the escape velocity becomes  $v_{esc}^2 = 2(1+\kappa^2)[\ln(r_1/r_*) + 1]$ . The value of  $v_{esc}$  is larger than the standard one [8] by a factor  $(1+\kappa^2)$ , so that  $\langle v_d^2 \rangle$  remains substantially smaller than  $v_{esc}^2$  for  $r_* \ll r_1$ . A particle that does not interact with the scalar field is bound only by the the potential  $\Phi$ . However, the scale of its velocity is set by  $v_c$ , so that again it cannot escape.

Our discussion supports the conclusion that the presence in galaxies of a scalar field related to dark energy does not have any disastrous phenomenological consequences. On the contrary, this scenario may be much more interesting for dark matter searches than the conventional one. Depending on the value of  $\kappa$ , the typical velocity of dark matter particles can exceed significantly the observed rotation velocity ( $\sim 220$  km/s for the Milky Way). As the density profile is not significantly modified, the estimated local energy density of dark matter remains the same as in the case with  $\kappa^2 = 0$ . It is  $\sim 0.4$  GeV/cm<sup>3</sup> for our neighborhood of the Milky Way. As a result the flux of dark matter particles towards a terrestrial detector is larger roughly by a factor  $(1+\kappa^2)^{1/2}$  relative to the  $\kappa^2 = 0$  case. A detailed study must take into account the motion of the Earth through our galaxy. However, the Earth velocity is of the order of  $v_c$ , and gives

only a modest correction. The cross section for the elastic scattering of halo particles by target nuclei through weak or strong interactions is largely independent of the particle velocity [9]. The leading effect of a non-zero value of  $\kappa$  is that the counting rates, that are proportional to the velocity, are increased by the factor  $(1 + \kappa^2)^{1/2}$ . This makes the dark matter easier to detect. Existing bounds on dark matter properties from direct searches can be extended to include the case of non-zero  $\kappa$ . The bound on the cross section for the interaction of dark matter with the material of the detector must be strengthened by  $(1 + \kappa^2)^{1/2}$ .

The allowed range of  $\kappa$  is limited by the observable implications of the model that describes the dark sector. Strong constraints arise from the measured fluctuations of the cosmic microwave background. The dependence of the mass of dark matter particles on an evolving scalar field during the cosmological evolution since the decoupling is reflected in the microwave background. The magnitude of the effect is strongly model dependent. In the models of ref. [4, 10], which assume an exponential dependence of the dark matter mass on the field, the observations result in the constraint  $\kappa^2 \lesssim 0.01$ . In the model of refs. [5, 6] the dark matter mass is a linear function of the field. The induced scalar interaction among dark matter particles is screened by an additional relativistic dark matter species. As a result, the model is viable even for couplings  $\kappa^2 \simeq 1$ . A similar mechanism is employed in ref. [11] in order to satisfy the observational constraints. In this model the interaction between dark matter and dark energy becomes important only during the recent evolution of the Universe.

It is reasonable to expect that the resolution of the coincidence problem through an interaction between dark matter and dark energy will have to rely on a coupling not significantly weaker than gravity. It seems unlikely that a coupling  $\kappa^2 \ll 1$  can lead to a cosmological evolution drastically different from that in the decoupled case. We expect the effects described in this letter to be natural consequences of any viable model that achieves the resolution of the coincidence problem through the coupling of dark matter to dark energy.

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